Wave: • an oscillation at a frequency

frequency of oscillation is fundamental

T = period ≡ time for 1 full oscillation

f = frequency ≡ how many oscillations per time

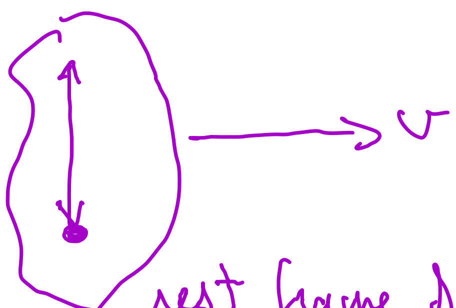
for 1 full oscillation: $\boxed{f = \frac{1}{T}}$

Traveling wave: oscillation moves with a velocity

oscillation is "transverse" to dir of motion

↕ ⟶ direction of motion

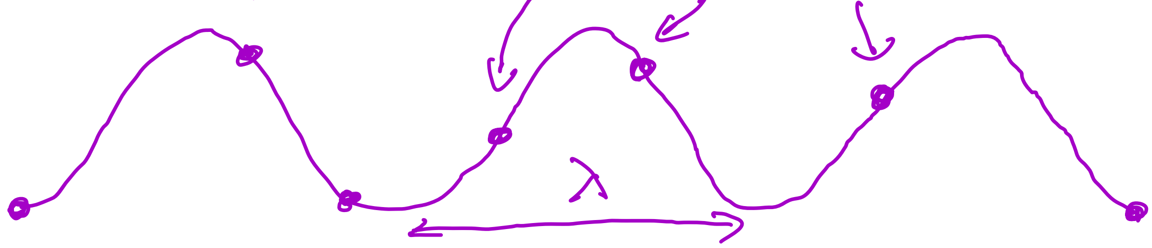
if oscillation is of a ball, it will trace out a sine wave when travelling



velocity of ball's rest frame

rest frame of the oscillating ball

snapshots!



wavelength: distance over which oscillation repeats

$$\text{velocity} = \frac{\text{dist}}{\text{time}} = \frac{\lambda}{T}$$

λ: dist for 1 repetition

T: time for 1 repetition

$$\boxed{v = \frac{\lambda}{T} = \lambda f}$$

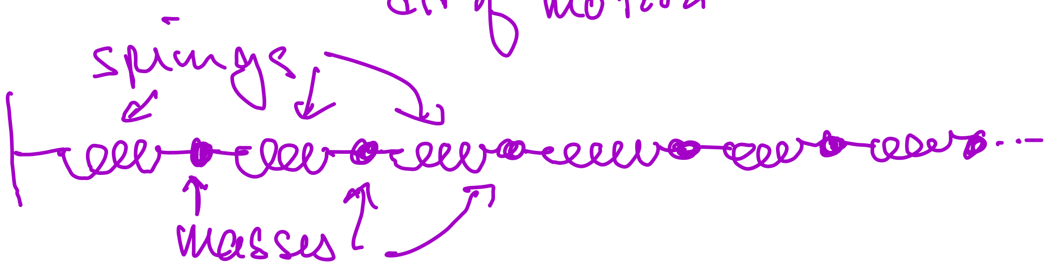
if ball velocity slows down, f stays the same!

ex: water waves are transverse waves (amplitude $\perp$ velocity)

EM waves are transverse $\left(\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0}\right)$

longitudinal waves: oscillation is along dir of motion

ex:

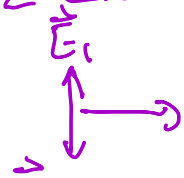


if you displace 1st ball (along axis of the spring) it will set up a longitudinal wave that will propagate along at some velocity

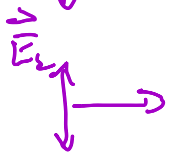
$\Rightarrow$ sound is a longitudinal pressure wave

Additions of waves

2 EM waves, oscillating $\vec{E}$ fields, along $\hat{x}$ dir



$$\vec{E}_1 = E_1 \hat{x} \cos(k_1 z - \omega_1 t)$$



$$\vec{E}_2 = E_2 \hat{x} \cos(k_2 z - \omega_2 t)$$

if they overlap → E fields add linearly

$$\vec{E}_{TOT} = \vec{E}_1 + \vec{E}_2$$

waves (oscillations) add linearly

⇒ <u>any</u> waves of <u>any</u> kind (not just EM waves)

this is the principle of superposition

⇒ in this chapter we are studying this for waves that have same freq & wavelength & even amplitude

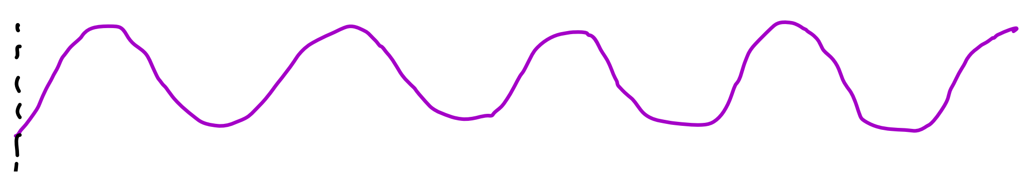
<u>But</u> they can have different phases

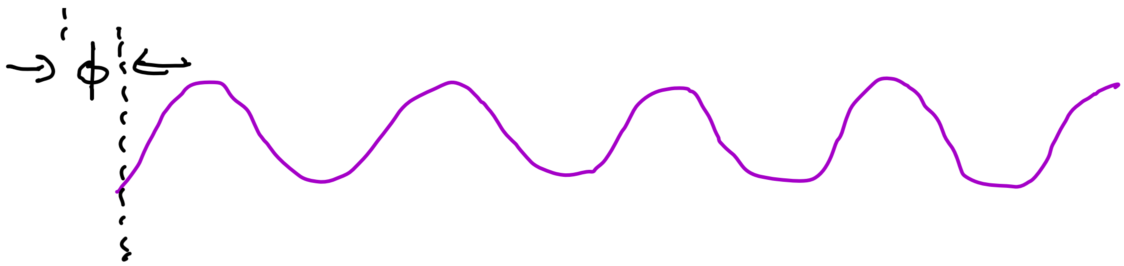
wave 1: $A(x) = A_0 \cos(kx)$ (snapshot at some time t)

$\Updownarrow$ same amp

" 2: $B(x) = A_0 \cos(kx + \phi)$

ϕ = phase diff between the 2 waves





ϕ is the <u>angular</u> phase diff

special cases:

$\phi = 0$: $A+B = 2A_0\cos(kx) \Rightarrow$ "constructive interference"

$\phi = \pi$: $A+B = A_0\cos(kx) + A_0\cos(kx+\pi)$

expand $\cos(kx+\pi) = \cos kx \overset{-1}{\cos\pi} - \sin kx \overset{0}{\sin\pi}$

$= -\cos kx$

so $A+B = A_0\cos(kx) - A_0\cos(kx) = 0$

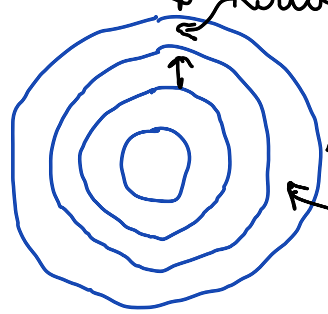
this is "destructive interference"

we are interested in

1. interference in 2 & 3 dimensions
2. condition for con & destructive interference in 2 & 3 dimensions

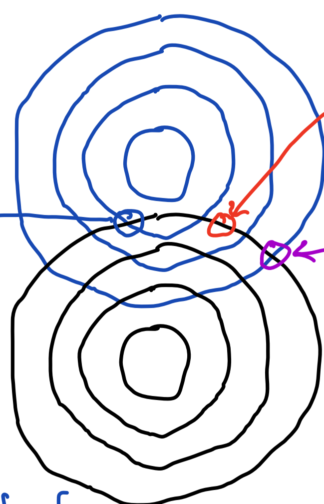
Interference in 2 dimensions

⇒ drop rock in water - waves propagate outward (concentric)



light waves are in 3-D but 2-D is sufficient to understand interference in 3-D

2 waves in 2D, same amplitude & wavelength



peak & trough overlap
⇒ destructive interference

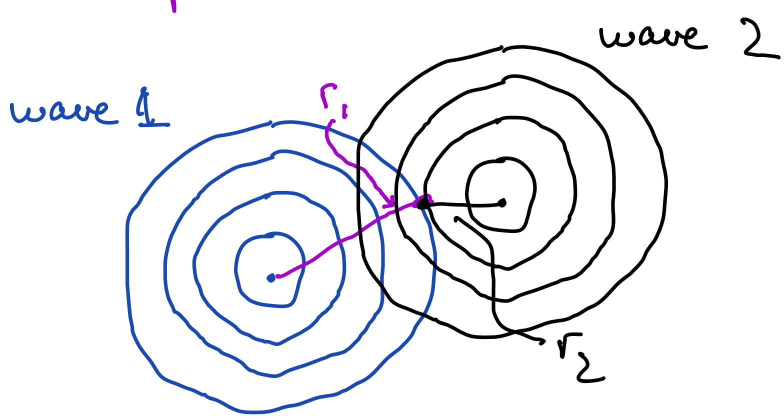
intersection of both peaks gives constructive interference
⇒ resulting amplitude is sum of each

constructive interference
⇒ 3^{rd} wavelength for black, 2^{nd} for blue

Condition for constructive interference:

- each wave goes a distance r from origin to a point

now overlap!



for constructive interference at any point, want each path length to be some multiple of wave length λ

$r_1 = n_1 \lambda$, $r_2 = n_2 \lambda$ $\quad n \equiv$ integer

or $\quad r_2 - r_1 \equiv \Delta r = n\lambda$ where $n \equiv$ integer

- so for constructive interference at a point:

$$\Delta r = n\lambda \quad \text{constructive}$$

- for destructive interference, one of the distances has to be $n\lambda + \frac{1}{2}\lambda$ (but only one, not both!)

$$\Delta r = (n+\tfrac{1}{2})\lambda \quad \text{destructive}$$

Phase difference

phase ϕ is the distance, in angle space 0-2π

so $\phi = 2\pi \frac{r}{\lambda}$ if $r = \lambda$, $\phi = 2\pi$

phase difference between 2 waves at pt of interest: $\Delta\phi = \phi_2 - \phi_1 = 2\pi\frac{(r_2 - r_1)}{\lambda} = \frac{2\pi}{\lambda}\Delta r$

can write $k = \frac{2\pi}{\lambda}$ "wave number"

so $\Delta\phi = k\Delta r$

condition for:

constructive interference: $\boxed{\Delta\phi = k \cdot n\lambda = 2\pi n}$

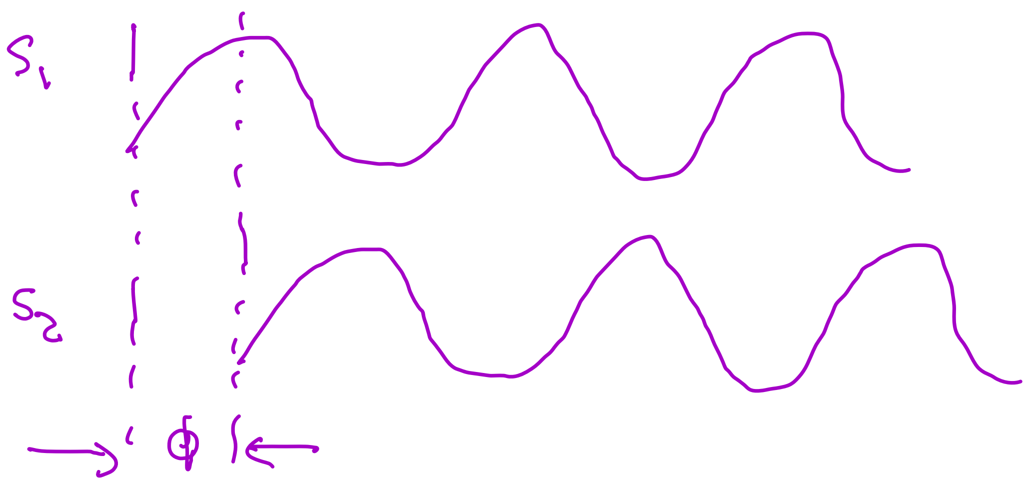
destructive " $\boxed{\Delta\phi = k(n+\tfrac{1}{2}\lambda) = 2\pi(n+\tfrac{1}{2})}$

Interference & intensity

→ take 2 waves, same amplitude & wavelength but with a constant phase difference ϕ

$$\left.\begin{aligned} S_1 &= A\cos(kx) \\ S_2 &= A\cos(kx+\phi) \end{aligned}\right\} k = \frac{2\pi}{\lambda}$$

ex:



add together: $S = S_1 + S_2$ principle of superposition

$$S = A\cos kx + A\cos(kx+\phi)$$

eg for EM wave, $E_1 = E_0 \cos kx$
$E_2 = E_0 \cos(kx+\phi)$

$$E = E_1 + E_2$$

intensity $I = \varepsilon_0 E^2 c$ for EM in general: intensity/power always comes from squaring wave function

for EM:

$E^2 = (E_1+E_2)^2$ ⟵

easiest method: subtract $\phi/2$ from each wave

$E_1 = E_0\cos(kx - \frac{\phi}{2})$ $E_2 = E_0\cos(kx+\phi/2)$

let $\theta = \phi/2$ to make it easier to calculate

$E_1 = E_0\cos(kx-\theta)$ $E_2 = E_0\cos(kx+\theta)$

$$E_1+E_2 = E_0(\cos(kx-\theta)+\cos(kx+\theta))$$

$$= \frac{E_0}{2}\mathrm{Re}\left(e^{i(kx-\theta)}+e^{-i(kx-\theta)}+e^{i(kx+\theta)}+e^{-i(kx+\theta)}\right)$$

$$= \frac{E_0}{2}\mathrm{Re}\left(e^{ikx}\underbrace{[e^{-i\theta}+e^{i\theta}]}_{2\cos\theta}+e^{-ikx}[e^{i\theta}+e^{-i\theta}]\right)$$

$$= E_0\cos\theta\,\mathrm{Re}(e^{ikx}+e^{-ikx})$$

$$= 2E_0\cos kx\cos\theta$$

$$= 2E_{1,2}\cos\theta \qquad \text{where } E_{1,2} = E_0\cos(kx)$$

$$= 2E_{1,2}\cos\left(\frac{\phi}{2}\right)$$

if $\phi = 0$ then $E = 2E_0\cos kx$ (constructive)

and $I_0 = \varepsilon_0 c E^2 = 4\varepsilon_0 c E_0^2\cos^2 kx$

for $\phi \neq 0$ then intensity $I = 4E_{1,2}^2 \cos^2 \phi/2 \cdot \varepsilon_0 C$

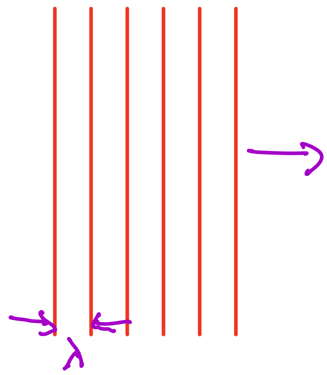
$$\boxed{I = I_0 \cos^2 \phi/2}$$

where ϕ = phase diff between the waves

and $I_0 = 4E^2$

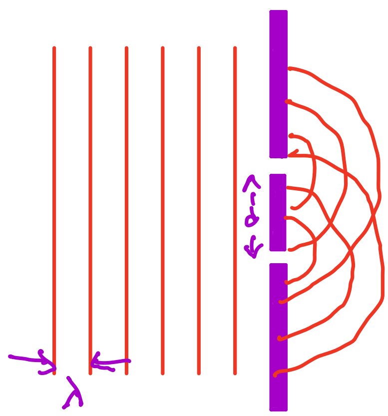
"2 slit" interference

take plane wave:



→ direction of propagation

can make source of 2 circular waves.



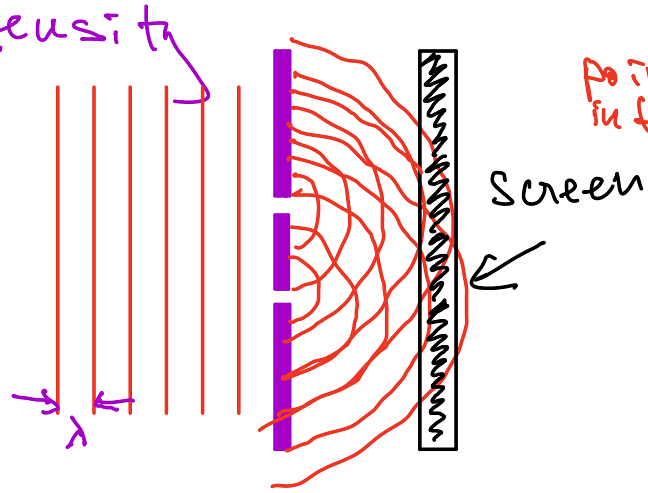
slits separated by distance "d"

each slit will let waves pass but will then spread out as if the slit were a pt source of waves

⇒ waves from slits will have same λ & φ ⇒ "coherent"

⇒ make slit width << wavelength of light, and d

now add a screen that can record resulting light intensity



point on screen where waves interfere constructively will have more intensity than when waves interfere destructively

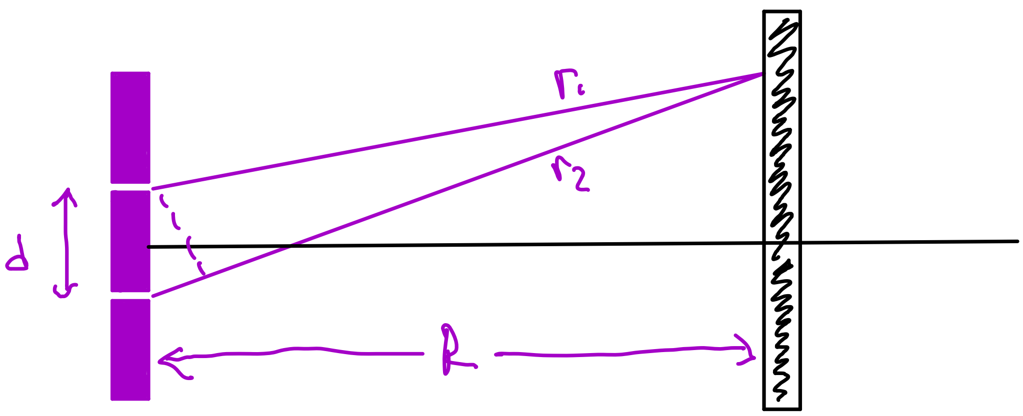
At the screen:

⇒ max intensity is where interference is <u>constructive</u>

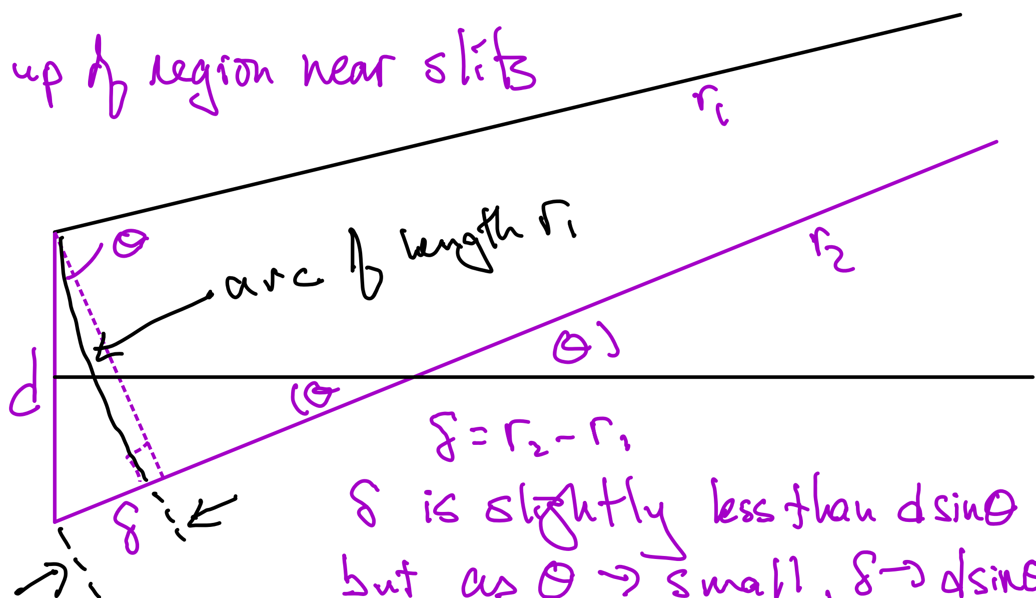
$\Delta r = n\lambda$

⇒ min intensity is where interference is <u>destructive</u>

$\Delta r = (n+\frac{1}{2})\lambda$



blow up of region near slits



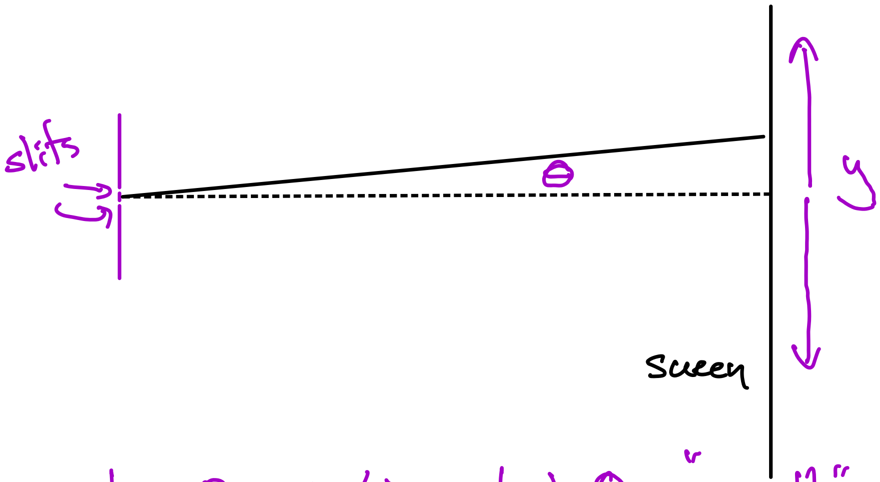
$\delta = r_2 - r_1$

δ is slightly less than $d\sin\theta$

but as $\theta \to$ small, $\delta \to d\sin\theta$

$\Delta r = \delta = d\sin\theta = n\lambda$ condition for constructive interference

similarly $\delta = d\sin\theta = (n+\frac{1}{2})\lambda$ condition for destructive now move screen so $R \gg d$



y measures position on screen

$\tan\theta = y/R$ but $\theta \sim$ "small" so $\tan\theta \sim \sin\theta$

$\sin\theta = y/R \Rightarrow y = R\sin\theta$

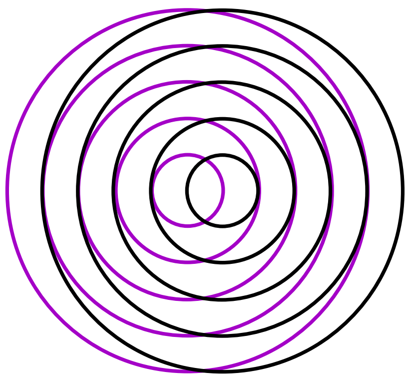
and $d\sin\theta = n\lambda$

so $y = \dfrac{Rn\lambda}{d}$ $\quad n = 0, \pm 1, \pm 2, \ldots$

this tells you where the maximum intensities from interference

ex: single radio transmitter transmits uniformly
in all directions

$\Rightarrow$ 2 transmitters separated by a distance d



by adding multiple transmitters close together,
can "beam" most of the energy along a more
narrow path. see:

http://www.physics.umd.edu/hep/drew/optics/antenna.html